MA / MSC MT - 03

December - Examination 2015

M.A. / M.Sc. Previous Mathematics Examination

Differential Equations, Calculus of Variations and Special Functions

Paper - MA / MSC MT - 03

Time : 3 Hours] **[Max. Marks :- 80**

Note : The question paper is divided into three sections A, B and C. Use of calculator is allowed in this paper.

Section - A 8 x 2 = 16

Note : Section 'A' contain 08 very short answer type questions. Examinees have to attempt all questions. Each question is of 02 marks and maximum word limit is 30 words.

1) (i) Write the value of $J_{1/2}(x)$

(ii) Define Elliptic Partial differential equation of second order.

(iii) Write Laplace equation in polar co-ordinates.

(iv) Define characteristic function of a boundary value problem.

(v) Define a isoperimetric problem.

(vi) Write Euler-Lagrange equation for stationary value of integral

$$I = \int_{x_1}^{x_2} f(x, y, y', y'')\, dx$$

(vii) Write Gauss's Hyper-geometric differential equation.

(viii) Write generating function for Hermite Polynomial.

Section - B 4 x 8 = 32

Note : Section 'B' contain 8 short answer type questions. Examinees will have to answer any four 04 questions. Each question is of 08 marks Examinees have to delimit each answer in maximum 200 words.

2) Find the general solution of Riccati's equation.

$$\frac{dy}{dx} = 2 - 2y + y^2$$

Whose one particular solution is $(1 + \tan x)$

3) Solve:

$$z^2 dx + (z^2 - 2yz)\, dy + (2y^2 - yz - zx)\, dz = 0$$

4) Solve the following strem Liouville problem.

$$y'' + \lambda y = 0,\ y'(-\pi) = 0,\ y'(\pi) = 0$$

5) Find the curve with fixed boundary revolves such that its rotation about x-axis generated minimal surface area.

6) If $|z| < 1$ and if $\mathrm{Re}(c) > \mathrm{Re}(b) > 0$ then prove that

$${}_2F_1(a, b, c, z) = \frac{\Gamma(c)}{\Gamma(b)\,\Gamma(c-b)} \int_0^1 t^{b-1}(1-t)^{e-b-1}(1-zT)^{-a} dt$$

7) Prove that

$$\frac{d}{dx}[{}_2F_1(a, b, c, x)] = \frac{ab}{c}\, {}_2F_1(a+1; b+1; c+1; x)$$

8) Prove that:

$$\frac{1+z}{z\sqrt{1-2xz+z^2}}-\frac{1}{z}=\sum_{n=0}^{\infty}(P_n+P_{n+1})z^n$$

9) Prove that:

$$(n+1)\,L_{n+1}(x)=(2n+1-x)\,L_n(x)-nL_{n-1}(x)$$

Section - C 2 x 16 = 32

Note : Section 'C' contain 04 Long answer type questions. Examinees will have to answer any two (02) questions. Each question is of 16 marks. Examinees have to delimit each answer in maximum 500 words.

10) (i) Solve $r+(a+b)s+abt=xy$ by Monge's method.

(ii) Solve by the method of separation of variables the PDE $4\frac{\partial u}{\partial t}+\frac{\partial u}{\partial x}=3u$ given that $u=3e^{-x}-e^{-5x}$ when $t=0$.

11) If z is a curve which is dependent on x & y and is twice differentiable in its domain D, and extremize the functional

$$I\,[z(x,y)]=\iint_D f(x,y,p,q)\,dx\,dy$$

Then prove that following equation must be satisfied

$$\frac{\partial f}{\partial z}-\frac{\partial}{\partial x}\left(\frac{\partial f}{\partial p}\right)-\frac{\partial}{\partial y}\left(\frac{\partial f}{\partial q}\right) \text{ where } p=\frac{\partial z}{\partial x} \text{ and } q=\frac{\partial z}{\partial y}$$

12) (i) Solve in series: $2x^2y''-xy'+(1-x^2)\,y=0$

(ii) Prove that : $x\,J'_n(x)=x\,J_{n-1}(x)-x\,J_n(x)$

13) (i) State and prove Rodrigues formula for Legendre polynomials.

(ii) Find the value of $J_5(x)$ in the terms of $J_0(x)$ and $J_1(x)$